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# Cancelling Jüttner distributions for space-like freeze-out

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**Abstract.** We study the freeze-out process of particles across a three-dimensional space-time hypersurface with space-like normal. The problem of negative contribution is discussed with respect to conservation laws, and a simple and practical new one-particle distribution for the post-FO side is introduced, the Cancelling Jüttner (CJ) distribution.

**PACS.** 25.75.-q Relativistic heavy-ion collisions – 25.75.Ld Collective flow – 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes

## 1 Introduction

Relativistic heavy-ion collisions are non-equilibrium relativistic many-body systems that can be described by different models, often based on the kinetic theory. The kinetic theory establishes a relationship between macroscopic and microscopic matter properties mostly by using a one-particle distribution function. Kinetic theories are able to describe dilute weakly interacting systems, thus expanding systems where the constituent particles gradually loose contact. Here to simplify such a process we will assume a freeze-out (FO) hypersurface in space-time, which can have either space- or time-like normal vector. We will use conservation laws of fluid dynamics to conserve energy, momentum and particle flow across FO hypersurface. We assume thermal equilibrium in pre-FO side where the matter is (or can be) in quark gluon plasma (QGP) phase. For the pre-FO side the Bag Model equation of state will be used which allows for supercooling and rapid hadronization afterwards. For the frozen-out matter (post-FO side) newly formed hadrons are considered, which are non-interacting with each other, and they are described by a one-particle distribution function. The invariant number of conserved particles (world-lines) crossing a surface element, $\mathrm{d}\sigma^\mu$, of the FO hypersurface is $\mathrm{d}N = N^\mu \mathrm{d}\sigma_\mu$. Thus, the total number of all particles crossing the FO hypersurface is $N = \int_S N^\mu \mathrm{d}\sigma_\mu$. From kinetic definition of the baryon current four-vector, $N^\mu$, we have

$$N^\mu = \int \frac{\mathrm{d}^3 p}{p^0} p^\mu f_{\mathrm{FO}}(\vec{r}, p; T, n, u^\mu). \tag{1}$$

Inserting it into the equation for the total number of particles leads to the Cooper-Frye formula [1]

$$E \frac{\mathrm{d}N}{\mathrm{d}^3 p} = \int f_{\mathrm{FO}}(\vec{r}, p; T, n, u^\mu) p^\mu \mathrm{d}\sigma_\mu, \tag{2}$$

where $f_{\mathrm{FO}}(\vec{r}, p; T, n, u^\mu)$ is the unknown post-FO phase space distribution of frozen out particles. The problem is to choose its form correctly, and to determine its parameters which satisfy all conservation laws! This was not done correctly in the original Cooper-Frye description [1]. The most used distribution for the time-like normal is the Jüttner distribution [2] (also called relativistic Boltzmann distribution):

$$f^{\mathrm{Jüttner}}(\vec{r}, p) = \frac{1}{(2\pi\hbar)^3} \exp\left(\frac{\mu(\vec{r}) - p^\mu u_\mu(\vec{r})}{T(\vec{r})}\right). \tag{3}$$

For the time-like case $p^\mu$ and $\mathrm{d}\sigma^\mu$ in the Cooper-Frye formula are both time-like vectors, thus $p^\mu \mathrm{d}\sigma_\mu > 0$, and the integrand of integral (2) is always positive. For the space-like normal vector case, $p^\mu \mathrm{d}\sigma_\mu$ can be both positive and negative. This situation is a problem, because the integrand in integral (2) may change sign, and this indicates that part of distribution contributes to a negative current, going back into the front, while the other part is coming out of the front. The improvement of the distribution function (3) was done by introducing the cut-Jüttner

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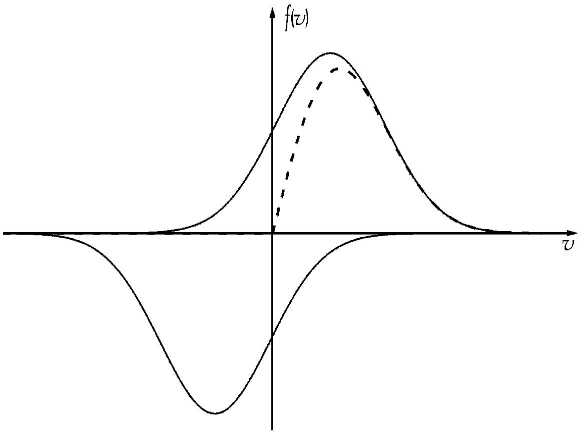


**Fig. 1.** Schematic view of making the Cancelling Jüttner distribution: in the rest frame of the front two Jüttner distributions (full curves), one with negative velocity, the other with positive velocity, are subtracted. The dashed curve is the CJ distribution.

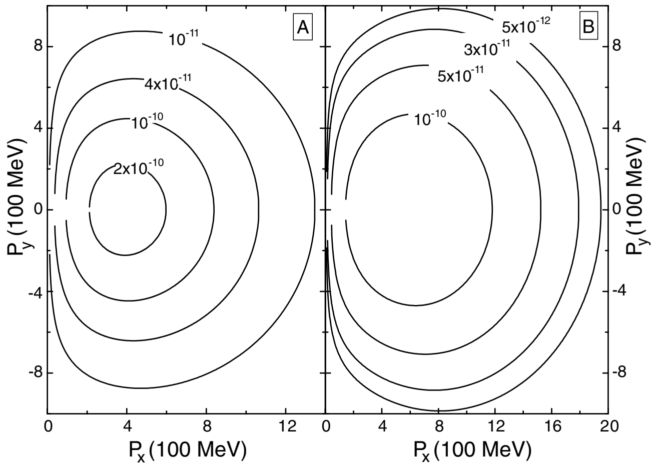


**Fig. 2.** The post-FO CJ distribution, $f_{\rm CJ}$, in the rest frame of the front (RFF). Baryon mass, $m = 1$ GeV, temperature, $T = 100$ MeV, density, $n = 0.17$ fm$^{-3}$ and velocity, $v = 0.3c$ in A, $v = 0.5c$ in B. The values of each contour are written on the curves. The CJ distribution resembles strongly the post-FO distribution obtained in kinetic theory, figs. 2 and 3 in ref. [5].

distribution [3,4], which makes $p^\mu d\sigma_\mu$ to be non-negative in the Jüttner distribution by multiplying it with the step function: $f^{\text{cut-Jüttner}} = \Theta(p^\mu d\sigma_\mu) f^{\text{Jüttner}}$. The cut-Jüttner distribution has solved the above-mentioned problem formally, but the lack of a real physical solution persisted. The cut-Jüttner distribution has an unphysical form: the distribution is sharply cut off which is mathematically satisfactory, but it is hardly possible to imagine a realistic physical process producing such a distribution. So, taking into account that this distribution describes physical particles, it has to be improved. A solution to this problem in kinetic theory was presented in ref. [5].

In the present work we will present a new, simple distribution, called the Cancelling Jüttner (CJ) distribution. It solves the problem of negative contributions in the Cooper-Frye formula, and it has a smooth physically realistic form. We worked out simple relations, which can be used for the calculation of measurables and can be compared with experimental results.

## 2 The Cancelling Jüttner distribution

The Cancelling Jüttner (CJ) distribution, $f_{\rm CJ}$, is defined by subtracting the ordinary Jüttner distribution (3) with negative velocity, $-v$, from the original Jüttner distribution, and multiplying the obtained result with the step function (fig. 1):

$$f_{\rm CJ} = \left(f_{\rm R}^{\text{Jüttner}} - f_{\rm L}^{\text{Jüttner}}\right)\Theta(p^\mu d\sigma_\mu) = \frac{\Theta(p^\mu d\sigma_\mu)}{(2\pi\hbar)^3}\left(\exp\left[\frac{\mu - p^\mu u_\mu^{\rm R}}{T}\right] - \exp\left[\frac{\mu - p^\mu u_\mu^{\rm L}}{T}\right]\right), \quad (4)$$

where $u_\mu^{\rm R} = (\gamma, \gamma v, 0, 0)$ and $u_\mu^{\rm L} = (\gamma, -\gamma v, 0, 0)$ in the rest frame of the front (RFF). The velocity parameter, $v$, of the CJ distribution is restricted to be positive. When $p^\mu d\sigma_\mu = 0$, the function vanishes at the front, even without step function $\Theta$. The role of the $\Theta(p^\mu d\sigma_\mu)$ part is just to eliminate the negative part of the distibution.

We chose this construction from two Jüttner distributions because: 1) It automatically includes the cut, and furthermore a smooth but rapid cut, thus the unrealistic profile of the earlier proposed cut-Jüttner distribution [4] is not present. 2) It resembles the distribution we obtained from a kinetic model quite well, in any case much better than the cut-Jüttner distribution. 3) The formulation is still analytic and not more complicated than the one arising from the cut-Jüttner distribution.

Calculations and results will be made in the rest frame of the front (RFF), where the surface element is $d\sigma_\mu = (0, 1, 0, 0)d\sigma$. In this frame, particles cannot propagate across the front in the negative $x$-direction. We assume that the pre-FO side is in thermal equilibrium. If the particle has passed the freeze-out layer it cannot scatter back. This layer is idealized as a front or hypersurface. In fig. 1, the schematic way of making the CJ distribution is shown. The distribution obtained this way (fig. 2) resembles strongly the one obtained numerically in a kinetic FO model [5]. In the following, we demonstrate the use of the Cancelling Jüttner distribution.

## 3 Conservation laws of fluid dynamics

The FO hypersurface is considered to be a discontinuity in space-time, thus instead of derivatives of the discontinuous variables we consider the changes explicitly. We will use the conservation laws of fluid dynamics, but instead of the differential form of continuity equations we have to use the commutator form:

$$[N^\mu d\sigma_\mu] = 0 \ \text{ and } \ [T^{\mu\nu} d\sigma_\mu] = 0, \quad (5)$$

where $[A] = A_1 - A_0$, $A_1$ is a post-FO quantity, and $A_0$ is the pre-FO quantity. Moreover, entropy across FO hypersurface must not decrease:

$$[S^\mu \mathrm{d}\sigma_\mu] \geq 0. \tag{6}$$

This condition does not provide additional information about the freezing-out matter, but it must be satisfied.

Now, our task is to find out the expressions for baryon current, energy-momentum tensor and entropy current in the post-FO side. Having these expressions, we will insert them into the equations of conservation laws (5), and then we will have a relationship between matter properties in pre- and post-FO sides. This calculation is straightforward for time-like FO, where both the pre- and post-FO sides can be characterized as perfect fluids with well-defined equation of state and energy-momentum tensor (see [6]). In the case of space-like FO with out-of-equilibrium post-FO distribution, this calculation has been demonstrated so far only for the cut-Jüttner distribution [3].

The post-FO baryon current, $N^\nu$, can be calculated inserting the post-FO distribution (4) into eq. (1). The energy-momentum tensor, $T^{\mu\nu}$, can be obtained in the same way, *i.e.*, inserting the post-FO distribution (4) into the following definition:

$$T^{\mu\nu} = \int \frac{\mathrm{d}^3 p}{p^0} p^\mu p^\nu f_{\mathrm{FO}}(\vec{r}, p; T, n, u^\mu)\,. \tag{7}$$

However, we do not have to calculate integral (1) nor (7), but instead we take the already calculated $N^\mu$, $T^{\mu\nu}$ and $S^\mu$ for the post-FO cut-Jüttner distribution in the reference frame of the gas (RFG), from [3]:

$$\begin{aligned}
N^0 &= \frac{\tilde{n}}{4}\left[vA + a^2 j[(1+j)K_2(a) - \mathcal{K}_2(a,b)] + j\frac{b^3 v^3}{3}e^{-b}\right],\\
N^x &= \frac{\tilde{n}}{8}\Big[(1-v^2)A - a^2 e^{-b}\Big],\\
T^{00} &= \frac{3\tilde{n}T}{2}\Big[j\frac{a^2}{2}\Big((1+j)[K_2(a) + \frac{a}{3}K_1(a)]\\
&\quad -\mathcal{K}_2(a,b) - \frac{a}{3}\mathcal{K}_1(a,b)\Big) + Bv\Big],\\
T^{0x} &= T^{x0} = \frac{3\tilde{n}T}{4}\left[\left(1-v^2\right)B - \frac{a^2}{6}(b+1)e^{-b}\right],\\
T^{xx} &= \frac{\tilde{n}T}{2}\left[j\frac{a^2}{2}[(1+j)K_2(a) - \mathcal{K}_2(a,b)] + v^3 B\right],\\
T^{yy} &= T^{zz} = \frac{3\tilde{n}T}{4}\left[v\left(1-\frac{v^2}{3}\right)B + \frac{ja^2}{3}\Big((1+j)K_2(a)\right.\\
&\quad \left. -\mathcal{K}_2(a,b)\Big) - \frac{va^2}{6}(b+1)e^{-b}\right],\\
S^0 &= \frac{\tilde{n}}{4}\left[\left(1-\frac{\mu}{T}\right)vA + 6vB + \left(1-\frac{\mu}{T}\right)a^2 j\big((1+j)K_2(a)\right.\\
&\quad \left. -\mathcal{K}_2(a,b)\big) + ja^2\big((1+j)K_1(a) - \mathcal{K}_1(a,b)\big)\right],\\
S^x &= \frac{\tilde{n}}{8}\left[(1-v^2)\left(1-\frac{\mu}{T}\right)A + 6(1-v^2)B\right.\\
&\quad \left. -a^2\left(2+b-\frac{\mu}{T}\right)e^{-b}\right],
\end{aligned}$$

where $j = \mathrm{sign}(v)$, $\tilde{n} = \pi T^3 e^{\mu/T}(\pi\hbar)^{-3}$, $\mu$ is the chemical potential of the pre-FO matter, $a = m/T$, $b = a/\sqrt{1-v^2}$, $v = \mathrm{d}\sigma_0/\mathrm{d}\sigma_x$, $A = (2+2b+b^2)e^{-b}$, $B = (1+b+b^2/2+b^3/6)e^{-b}$ and

$$\mathcal{K}_n(a,b) = \frac{2^n(n!)}{(2n)!}a^{-n}\int_b^\infty \mathrm{d}x(x^2-a^2)^{n-1/2}e^{-x},$$

*i.e.* $K_n(a) = \mathcal{K}_n(a,a)$. All other components of $T^{\mu\nu}, N^\mu$ and $S^\mu$ vanish.

The derivation of $N^\mu_{\mathrm{CJ}}$ and $T^{\mu\nu}_{\mathrm{CJ}}$ will be presented in detail in the mass-zero limit in the next section. However, the way of deriving these quantities is the same for both cases: with mass-zero limit, or without. The results of $N^\mu_{\mathrm{CJ}}$, $T^{\mu\nu}_{\mathrm{CJ}}$ and $S^\mu_{\mathrm{CJ}}$ for finite-mass $m$, in RFF are:

$$\begin{aligned}
N^0_{\mathrm{CJ}} &= \gamma\frac{\tilde{n}}{4}\Big[3Av - va^2e^{-b} - Av^3 + 2a^2\Delta_2\Big],\\
N^x_{\mathrm{CJ}} &= \gamma\frac{\tilde{n}}{2}\left[a^2vK_2(a) + j\frac{b^3v^4}{3}e^{-b}\right],\\
T^{00}_{\mathrm{CJ}} &= 3\tilde{n}T\gamma^2\left[j\frac{a^2}{2}\left(\frac{a}{3}\Delta_1 + \left(1+\frac{v^2}{3}\right)\Delta_2\right)\right.\\
&\quad \left. +B\left(\frac{v^5}{3} - 4v^3 + 5v\right) - va^2(b+1)e^{-b}\right],\\
T^{x0}_{\mathrm{CJ}} &= T^{0x}_{\mathrm{CJ}} = \frac{\tilde{n}T\gamma^2a^2}{2}\Big[4vK_2(a) + avK_1(a)\Big],\\
T^{xx}_{\mathrm{CJ}} &= 3\tilde{n}T\gamma^2\left[j\frac{a^2}{2}\left(v^2\left(\Delta_2 + \frac{a}{3}\Delta_1\right) + \frac{1}{3}\Delta_2\right)\right.\\
&\quad \left. +B\left(-\frac{8}{3}v^3 + 4v\right) + a^2v^2(b+1)e^{-b}\right],\\
S^0_{\mathrm{CJ}} &= \gamma\frac{\tilde{n}}{4}\left[(3v-v^3)\left(\left(1-\frac{\mu}{T}\right)A + 6B\right)\right.\\
&\quad \left. +2a^2j\left(\left(1-\frac{\mu}{T}\right)\Delta_2 + \Delta_1\right) - a^2v\left(2+b-\frac{\mu}{T}\right)e^{-b}\right],\\
S^x_{\mathrm{CJ}} &= \gamma\frac{\tilde{n}}{2}\left[a^2K_2(a)v\left(1-\frac{\mu}{T}\right) + a^2K_1(a)v\right],
\end{aligned}$$

where $\Delta_i \equiv K_i(a) - \mathcal{K}_i(a,b)$, $i = 1, 2$ [1]. All other components vanish. From these expressions, one obtains properties of the matter on the post-FO side of the FO front.

## 4 Calculating the relationship of matter properties

Now we will study in more detail the change of matter properties across the FO hypersurface with space-like normal, having local equilibrium in pre-FO side, and non-equilibrated matter on the post-FO side. In order to connect both sides of the FO hypersurface, we will use conservation laws of hydrodynamics.

[1] When $v = 0$, *i.e.* $\Delta_i = 0$, all components of $T^{\mu\nu}$ and $N^\mu$ equal zero.

We are taking baryon current and energy momentum tensor for the cut-Jüttner distribution in the RFG with mass-zero limit[2]. Such an approximation is possible when energies are high compared to the masses of particles. From [3] $N^{\mu}_{\mathrm{RFG}}$ and $T^{\mu\nu}_{\mathrm{RFG}}$ read as

$$\begin{aligned}
N^0_{\mathrm{RFG}} &= \tilde{n}\frac{v+1}{2}, \qquad N^x_{\mathrm{RFG}} = \tilde{n}\frac{1-v^2}{4},\\
T^{00}_{\mathrm{RFG}} &= 3\tilde{n}T\frac{v+1}{2}, \quad T^{0x}_{\mathrm{RFG}} = 3\tilde{n}T\frac{1-v^2}{4},\\
T^{xx}_{\mathrm{RFG}} &= \tilde{n}T\frac{v^3+1}{2}, \quad T^{zz}_{\mathrm{RFG}} = T^{yy}_{\mathrm{RFG}} = \frac{T^{00}_{\mathrm{RFG}} - T^{xx}_{\mathrm{RFG}}}{2}.
\end{aligned}$$

To get the post-FO baryon current and energy-momentum tensor for the CJ distribution in the rest frame of the front (RFF), we are making a Lorentz transformation from the RFG to the RFF of the components of the cut-Jüttner distribution. Thus, we get $N^{\mu}_{\mathrm{RFF}}$ and $T^{\mu\nu}_{\mathrm{RFF}}$ for the cut-Jüttner distribution in mass-zero limit:

$$\begin{aligned}
N^{\mu}_{\mathrm{RFF}} &= \frac{\tilde{n}\gamma}{4}(-v^3+3v+2, v^2+2v+1, 0, 0),\\
T^{00}_{\mathrm{RFF}} &= \frac{\gamma^2\tilde{n}T}{4}(v^5-6v^3+2v^2+12v+6),\\
T^{0x}_{\mathrm{RFF}} &= T^{x0}_{\mathrm{RFF}} = \frac{\gamma^2\tilde{n}T}{4}(-v^4+6v^2+8v+3),\\
T^{xx}_{\mathrm{RFF}} &= \frac{\gamma^2\tilde{n}T}{2}(v^3+3v^2+3v+1), \quad T^{yy}_{\mathrm{RFF}} = T^{zz}_{\mathrm{RFF}} = T^{yy}_{\mathrm{RFG}}.
\end{aligned}$$

The energy-momentum tensor components $T^{yy}$ and $T^{zz}$ are not changing, because our reference frame is chosen in such a way that the Lorentz transformation is influencing only time and one spatial component $x$. All other energy-momentum tensor components are equal to zero.

Any of the components presented above for the CJ distribution, for example, $N^0_{\mathrm{CJ}}$ in the RFF frame, can be found in such a way:

$$N^0_{\mathrm{CJ}}(v) = N^0_{\mathrm{RFF}}(v) - N^0_{\mathrm{RFF}}(-v).$$

This follows from the definition of the CJ distribution function, eq. (4). Thus, the components of the baryon current of the CJ distribution take the form

$$N^{\mu}_{\mathrm{CJ}} = \tilde{n}\gamma\left(\frac{-v^3+3v}{2}, v, 0, 0\right), \tag{8}$$

and the components of the energy-momentum tensor:

$$T^{00}_{\mathrm{CJ}} = \tilde{n}T\gamma^2(v^5-3v^3+6v), \quad T^{0x}_{\mathrm{CJ}} = T^{x0}_{\mathrm{CJ}} = 4\tilde{n}T\gamma^2 v, \tag{9}$$

$$T^{xx}_{\mathrm{CJ}} = \tilde{n}T\gamma^2(v^3+3v), \quad T^{yy}_{\mathrm{CJ}} = T^{zz}_{\mathrm{CJ}} = \tilde{n}T\frac{-v^3+3v}{2}. \tag{10}$$

Now we can introduce flow, particle density, energy density and entropy density for the frozen-out matter. The four-flow of the particles, $u_{\mu}$, can be described in different ways: i) Eckart's definition:

[2] Notations of baryon current, energy-momentum tensor and entropy current with mass-zero limit and without are the same; from now on, $m = 0$ is assumed.

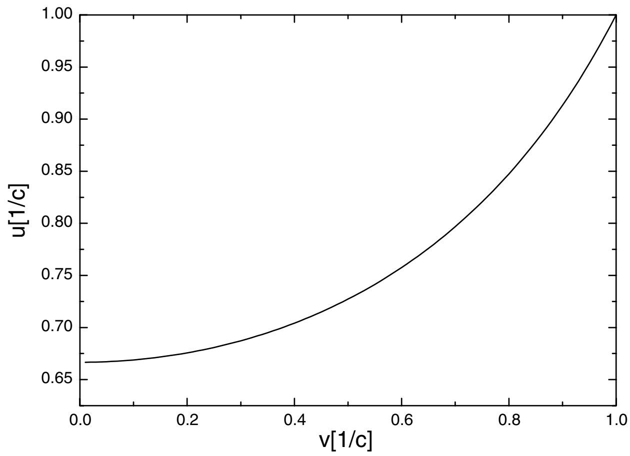


**Fig. 3.** Eckart's flow velocity, $u$, of the frozen-out particles, described by CJ distribution in RFF, *versus* the velocity parameter of the distribution, $v$.

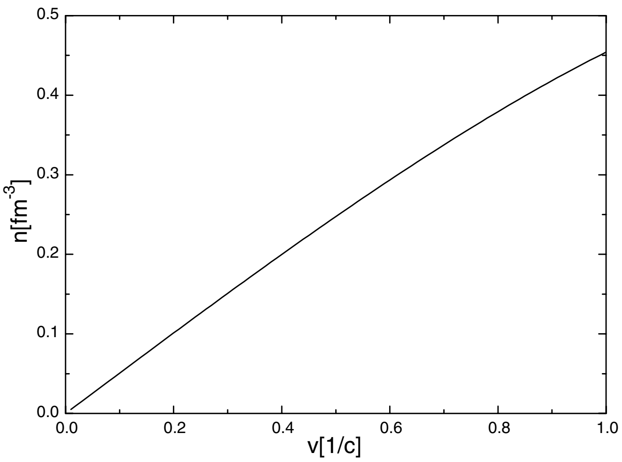


**Fig. 4.** The proper density of the frozen-out particles, $n$, described by CJ distribution, *versus* the velocity parameter of the distribution, $v$, with fixed chemical potential, $\mu = 10$ MeV, and temperature *parameter*, $T = 100$ MeV.

$$u^{\mu}_{\mathrm{flow,E}} = \frac{N^{\mu}_{\mathrm{CJ}}}{\sqrt{N^{\nu}_{\mathrm{CJ}} N_{\nu,\mathrm{CJ}}}}, \tag{11}$$

where the flow $u^{\mathrm{E}}_{\mu}$ is tied to conserved particles. ii) Landau's definition:

$$u^{\mu}_{\mathrm{flow,L}} = \frac{T^{\mu\nu}_{\mathrm{CJ}} u_{\nu}}{\sqrt{u_{\rho} T^{\rho\nu}_{\mathrm{CJ}} u_{\nu}}}, \tag{12}$$

where the flow is tied to energy flow. In case of Cancelling Jüttner distribution, these two definitions do not give an identical result, because the CJ distribution is not an equilibrium distribution. In further calculation, let us use Eckart's definition of the flow. Having the expression for the flow, we can obtain other macroscopic quantities. The invariant scalar density $n$ is

$$n = N^{\mu}_{\mathrm{CJ}} u^{\mathrm{E}}_{\mu}. \tag{13}$$

The energy density, $e$, and the entropy density, $s$, read as

$$e^{\mathrm{E}} = u^{\mathrm{E}}_{\mu} T^{\mu\nu} u^{\mathrm{E}}_{\nu}, \quad s^{\mathrm{E}} = S^{\mu} u^{\mathrm{E}}_{\mu}.$$

As we can see in fig. 3, where the flow velocity is plotted *versus* the velocity parameter of the CJ distribution, Eckart's flow has the minimum value $0.66c$ in the mass-zero limit. Figure 4 shows the post-FO density, $n^{\mathrm{E}}$, dependence on the velocity parameter, $v$.

## 5 Freeze-out from QGP

We must find expressions for the $N_0^\mu$ and $T_0^{\mu\nu}$ in the pre-FO side to use the conservation laws (5) across the surface element $\mathrm{d}\sigma_\mu$. For perfect fluids the energy-momentum tensor (in any reference frame) can be written as [6]

$$T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}, \tag{14}$$

where $e$ is the energy density and $p$ is pressure in the local rest (LR) frame. For the baryon current, we use Eckart's definition for the flow: $u^\mu = N^\mu_{(0)}/\sqrt{N^\nu_{(0)}N_\nu^{(0)}}$. From this definition there follows:

$$N^\mu_{(0)} = n_0 u^\mu_{(0)}, \tag{15}$$

where $u^\mu_{(0)} = (1,0,0,0)$ and the invariant scalar, $n_0 = \sqrt{N^\mu_{(0)}N_\mu^{(0)}}$, is the particle density.

To determine the parameters of the CJ distribution, we will use the conservation laws introduced by eq. (5). By using the conservation law for the four-current $[N^\mu\,\mathrm{d}\sigma_\mu] = N^\mu_{\mathrm{CJ}}\mathrm{d}\sigma_\mu - N^\mu_{(0)}\mathrm{d}\sigma_\mu^{(0)} = 0$, where $\mathrm{d}\sigma_\mu^{(0)} = \gamma_0(v_0,1,0,0)$ in the pre-FO local rest frame, and $\mathrm{d}\sigma_\mu = (0,1,0,0)$ in the RFF, we get the equation

$$\tilde{n}\gamma v - n_0\gamma_0 v_0 = 0. \tag{16}$$

Note that post- and pre-FO expressions have to be in the same reference frame, in order to use conservation laws correctly. Nevertheless, $N_0^\mu$ and $\mathrm{d}\sigma_\mu^{(0)}$ do not have to be transformed to the RFF, because the baryon number crossing the surface is an invariant scalar. However, the energy-momentum current, $T^{\mu\nu}\mathrm{d}\sigma_\mu$, is not an invariant scalar, thus it must be Lorentz-transformed to the RFF. After the transformation, $T^{\mu\nu}$ gets the form

$$T^{\mu\nu}_{(0)\mathrm{RFF}} = \begin{pmatrix} \gamma_0^2(e_0+v_0^2p_0) & \gamma_0^2(-e_0v_0-p_0v_0) & 0 & 0 \\ \gamma_0^2(-e_0v_0-p_0v_0) & \gamma_0^2(e_0v_0^2+p_0) & 0 & 0 \\ 0 & 0 & p_0 & 0 \\ 0 & 0 & 0 & p_0 \end{pmatrix}.$$

Using the conservation laws for the energy-momentum tensor, $[T^{\mu\nu}\ \mathrm{d}\sigma_\mu] = T^{\mu\nu}_{\mathrm{CJ}}\mathrm{d}\sigma_\mu - T^{\mu\nu}_{(0)}\mathrm{d}\sigma_\mu = 0$, we obtain two more equations:

$$4\gamma^2\tilde{n}Tv - \gamma_0^2\left(v_0e_0 + v_0p_0\right) = 0\,, \tag{17}$$

$$\gamma^2\tilde{n}T(v^3+3v) - \gamma_0^2\left(v_0^2e_0 + p_0\right) = 0\,. \tag{18}$$

Entropy should not decrease, $[S^\mu\mathrm{d}\sigma_\mu] = S^\mu_{\mathrm{CJ}}\mathrm{d}\sigma_\mu - S^\mu_{(0)}\mathrm{d}\sigma_\mu^{(0)} \geq 0$, this leads to the condition:

$$\tilde{n}\gamma v\left(1-\frac{\mu}{T}\right) - s_0\gamma_0v_0 \geq 0, \tag{19}$$

which must be satisfied.

The Bag Model Equation of State (EoS) for quark gluon plasma is used to describe the pre-FO matter state. It is assumed that quarks and gluons exist in perturbative vacuum, where plasma contains $2(N_c^2-1)$ gluons and $2N_cN_f$ quarks ($N_c$ and $N_f$ are the number of colors and number of flavors) [7]. The Bag Model EoS is based on Stefan-Boltzmann EoS, including a bag constant $B$. From [6], pre-FO quantities are expressed as

$$\begin{aligned} e_0 &= \left(\frac{37}{30}\pi^2T_0^4 + \frac{1}{3}\mu^2T_0^2 + \frac{1}{54\pi^2}\mu^4 + \Lambda_B^4\right)\frac{1}{(\hbar c)^3}\,, \\ n_0 &= \frac{2}{9}\left(\mu^2T_0^2 + \frac{1}{9\pi^2}\mu^3\right)\frac{1}{(\hbar c)^3}\,, \\ s_0 &= \left(\frac{74}{45}\pi^2T_0^3 + \frac{2}{9}\mu^2T_0^2\right)\frac{1}{(\hbar c)^3}\,, \\ p_0 &= \frac{e_0}{3} - \frac{4}{3}B, \quad \text{where} \quad B = \frac{\Lambda_B^4}{(\hbar c)^3}\,. \end{aligned}$$

Here $\mu$ is the baryon chemical potential.

Using the above and equations (16)-(18), we can evaluate how post-FO matter properties depend on the pre-FO side matter properties. It is enough to fix four quantities in the Bag Model EoS to obtain the properties of the post-FO matter. To describe the pre-FO side we will use initial temperature, $T_0$, bag constant, $\Lambda_B$, initial baryon density, $n_0$, and initial velocity, $v_0$, to have pre-FO energy density, pressure, entropy density and baryon chemical potential.

Nevertheless, the CJ distribution has a negative aspect: not for all initial values it is possible to calculate post-FO matter parameters. There are two reasons: 1) Entropy must not decrease. 2) The maximum of the Jüttner distribution function must be on the positive velocity side. When the maximum of the Jüttner distribution is at velocity zero ($f_{\text{Jüttner}}(v=0)|_{\mathrm{RFF}} = \max f_{\text{Jüttner}}(v)|_{\mathrm{RFF}}$), the CJ distribution is a "zero" function ($f_{\mathrm{CF}}(v){=}0, \forall\ v$). This problem depends on how pre-FO properties such as velocity, $v_0$, density, $n_0$, bag constant, $\Lambda_B$, and initial temperature, $T_0$, are chosen.

The boundary conditions for pre-FO velocity, $v_0$ and density, $n_0$, with fixed bag constant, $\Lambda_B$, and initial temperature, $T_0$, are shown in fig. 5. Such conditions for the pre-FO matter, for the values which are to the right with respect to the curves in fig. 5, have to be satisfied for the CJ distribution because we are dealing with a FO hypersurface with space-like normal. The shape of the curves is influenced by the way of making the CJ distribution, *i.e.* when the post-FO velocity becomes imaginary. The cutoff of the curves is influenced by the entropy condition. In order to have perspicuity, we compare two curves: one with initial temperature $T_0 = 60$ MeV, another with $T_0 = 100$ MeV ($\Lambda_B = 200$ MeV for both curves). Having higher temperature, the density (or velocity) can be lower than in the lower-temperature case. On the other hand, to ensure the entropy condition, the density in the higher-temperature case must be higher (more than $0.5\ \mathrm{fm}^{-3}$) than in the case of lower temperature, where it must be just more than $0.1\ \mathrm{fm}^{-3}$. Temperature is influencing the length of the curve and moves it to the

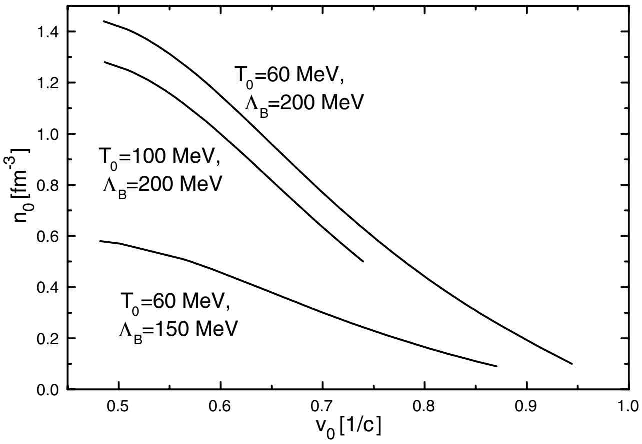


**Fig. 5.** The boundaries of the pre-FO density, $n_0$, and velocity, $v_0$, to have correct post-FO distribution function, $f_{\mathrm{CJ}}$, in the rest frame of the front (RFF) with fixed initial temperature, $T_0$, and bag constant, $\Lambda_B$. The CJ distribution is applicable for initial conditions for the values which are to the right with respect to the curves, *i.e.* for large flow velocities in RFF.

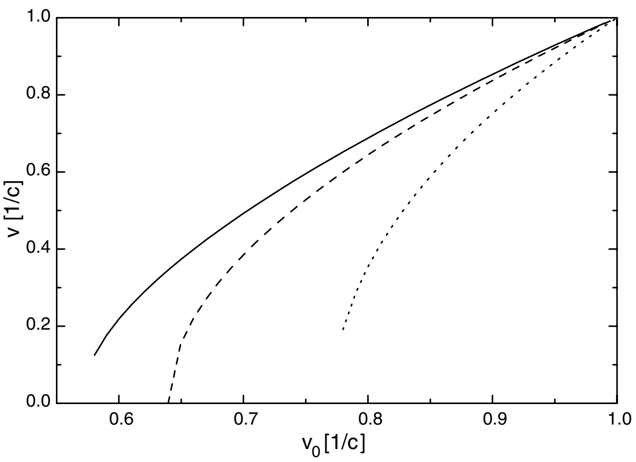


**Fig. 6.** Change of the velocity parameter across the FO hypersurface, in RFF, described by the CJ distribution for the different initial parameters: 1) $n_0 = 0.5$ fm$^{-3}$, $\Lambda_0 = 150$ MeV, full curve; 2) $n_0 = 1$ fm$^{-3}$, $\Lambda_0 = 200$ MeV, dashed curve; 3) $n_0 = 0.2$ fm$^{-3}$, $\Lambda_0 = 150$ MeV, dotted curve; and $T_0 = 60$ MeV for all curves.

left upon increasing, and bag constant is changing the gradient of the curve in the $(v_0, n_0)$-plane.

Now, having boundary conditions, we can start calculating matter properties of the post-FO side. From equations (16)-(18) we get one equation for the post-FO velocity parameter:

$$v = \sqrt{\frac{4(v_0^2 e_0 + p_0)}{v_0(e_0 + p_0)} - 3}.$$

Note that post-FO velocity, $v$, temperature, $T$, and density, $\tilde{n}$, are not physical quantities —they are parameters of the CJ distribution. As we can see in fig. 6, the post-FO velocity parameter is all the time smaller than the pre-FO velocity.

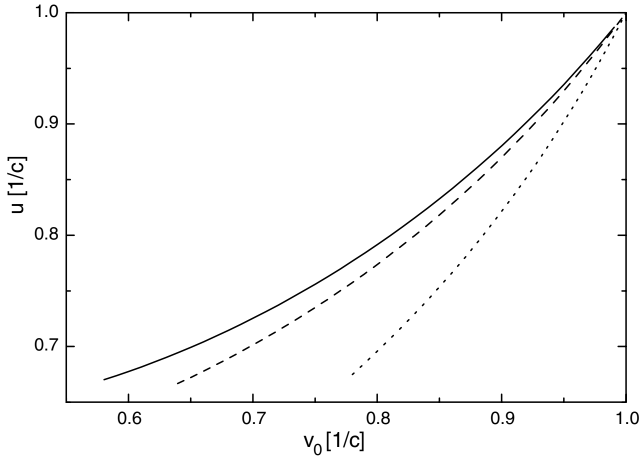


**Fig. 7.** The dependence of the post-FO baryon flow on the pre-FO velocity for the different initial parameters: 1) $n_0 = 0.5$ fm$^{-3}$, $\Lambda_0 = 150$ MeV, full curve; 2) $n_0 = 1$ fm$^{-3}$, $\Lambda_0 = 200$ MeV, dashed curve; 3) $n_0 = 0.2$ fm$^{-3}$, $\Lambda_0 = 150$ MeV, dotted curve; and $T_0 = 60$ MeV for all curves.

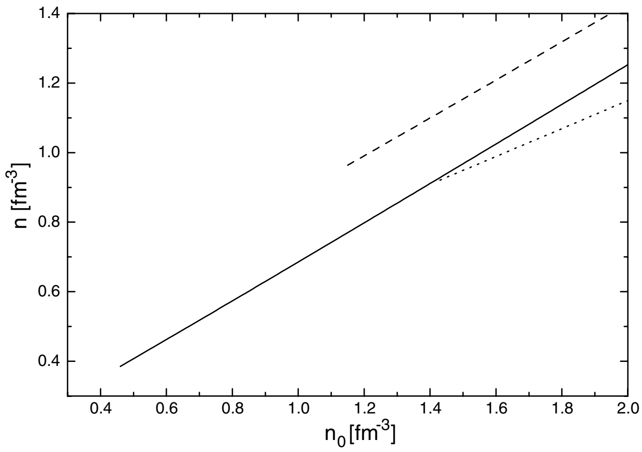


**Fig. 8.** The change of the baryon density across the FO hypersurface for different initial parameters: 1) $v_0 = 0.6c$, $\Lambda_0 = 150$ MeV, full curve; 2) $v_0 = 0.6c$, $\Lambda_0 = 200$ MeV, dashed curve; 3) $v_0 = 0.5c$, $\Lambda_0 = 200$ MeV, dotted curve; and $T_0 = 60$ MeV for all curves.

The post-FO flow velocity, $u_{\mathrm{flow}}$, is calculated using Eckart's definition of the flow using formula (11). Figure 7 shows the post-FO baryon flow velocity dependence on the pre-FO velocity, $v_0$. We observe that the flow does not decrease in the same way as the post-FO velocity parameter, $v$. Only in the case of low initial density ($n_0 = 0.2$ fm$^{-3}$ in fig. 7), we have decrease of velocity going from pre- to post-FO side. To calculate the final baryon density, we are using eq. (13), which can be rewritten for the case of Eckart's flow:

$$n = \sqrt{N^{\mu}_{\mathrm{CJ}} N_{\mu,\mathrm{CJ}}}.$$

From the results presented in fig. 8, it is seen that the baryon charge density in the post-FO side is decreasing compared to the density in the pre-FO side. The two lines are parallel to each other if initial velocity, $v_0$, and initial temperature, $T_0$, are the same.

## 6 Conclusions

We have studied the problem of negative contribution in the Cooper-Frye formula (2) for the 3-dimensional hypersurface with space-like normal. The Cancelling Jüttner distribution function was suggested as a solution for this problem. We have showed the applicability and properties of the CJ distribution by using the Bag Model equation of state for QGP and conservation laws of hydrodynamics across the FO hypersurface.

The CJ distribution solves the problem of negative contribution in the Cooper-Frye formula and has a physical form unlike the cut-Jüttner distribution. In future studies, one should analyze if the conditions of applicability of CJ distribution are satisfied in space-like hypersurfaces obtained from hydrodynamical calculations.

It will be interesting to see how large corrections this procedure will give to the results obtained from simple Cooper-Frye description.

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